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Anomaly of $(2, 0)$ Theories

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ABSTRACT

We compute gravitational and axial anomaly for D-type $(2, 0)$ theories realized on N pairs of coincident M5-branes at R^5/Z_2 orbifold fixed point. We first summarize work by Harvey, Minasian, and Moore on A-type $(2, 0)$ theories, and then extend it to include the effect of orbifold fixed point. The net anomaly inflow follows when we further take into account the consistency of T^5/Z_2 M-theory orbifold. We deduce that the world-volume anomaly is given by $N\mathcal{J}_8 + N(2N - 1)(2N - 2)p_2/24$ where \mathcal{J}_8 is the anomaly polynomial of a free tensor multiplet and p_2 is the second Pontryagin class associated with the normal bundle. This result is in accord with Intriligator's conjecture.

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1 Introduction

One of more mysterious results from study of string theory, is existence of nontrivial quantum theories in six dimensions. One class of these are known simply as $(2, 0)$ theories, which are supposed to be a non-Abelian generalization of free noninteracting tensor theories. Furthermore, there are three different types of $(2, 0)$ theories, classified by the ubiquitous ADE classification. One way [1] of realizing these theories is to consider type IIB compactification on an ALE space that asymptotes to R^4/Γ where Γ is one of finite subgroups of $SU(2)$, and take a small coupling limit of the string theory, while collapsing the cycles in the ALE spaces appropriately. Chiral 2-form tensors arise from chiral 4-form tensor of type IIB theories, while D3-branes wrapped on the collapsing cycles provide “charged” degrees of freedom that are necessary to complete the $(2, 0)$ theories. The ADE classification of the $(2, 0)$ theories follows from ADE classification of Γ .

In part because these theories cannot be written down in terms of usual path integral of local fields, very little is known about them. On the other hand, the question of gravitational and axial anomaly is inherently topological, and should be accessible without detailed knowledge of the theory. Indeed the question of gravitational and axial anomaly of these theories has a close tie to various topological terms in M-theory, in much the same way that the anomaly structure of heterotic string theory was vital in understanding topological aspects of M-theory [3, 4]. For the purpose, one considers N coincident M5 branes [5] in the limit of divergent Planck scale. In the flat space-time background, the tensor multiplets living on the M5 branes, along with degrees of freedom from open membranes suspended between adjacent M5, constitute A_{N-1} $(2, 0)$ theory. To produce D -type, we replace the flat space-time with an orbifold $R^{1+5} \times R^5/Z_2$, and put M5’s at the origin.

In this latter context, anomaly of $(2, 0)$ theories can be computed indirectly by asking what is the anomaly inflow from bulk onto M5 brane system; the total anomaly should vanish since we expect M-theory with M5 brane to be self-consistent. For the case of a single M5 brane in flat spacetime, Freed, Harvey, Minasian and Moore (FHMM) [6] observed that a particular deformation of the cubic Chern-Simon term of M-theory,

$$\int C \wedge dC \wedge dC, \tag{1}$$

appears naturally and seems necessary to cancel a single M5 brane anomaly [7] com-

pletely.

Subsequent extrapolation to many coincident M5's, due to Harvey, Minasian and Moore (HMM) [8] allows a simple computation of the anomaly polynomial of A_{N-1} theory. Eleven dimensional Lorentz group descends down to the six-dimensional Lorentz group and the axial $SO(5)$ group, each becoming structure groups of tangent and normal bundles of the M5's, respectively. In this unified Language, the total anomaly polynomial was argued to be,

$$N\mathcal{J}_8 + (N^3 - N) \frac{p_2(\mathcal{N})}{24}, \quad (2)$$

where \mathcal{J}_8 is the one-loop anomaly polynomial of a single $(2, 0)$ tensor multiplet, while p_2 is the second Pontryagin class of the normal bundle.

In this paper, we compute the anomaly polynomial of for D-type $(2, 0)$ theories by computing net anomaly inflow from bulk onto N pairs of coincident M5 branes at R^5/Z_2 orbifold fixed point. We will argue that the anomaly polynomial in this case is,

$$N\mathcal{J}_8 + N(2N - 1)(2N - 2) \frac{p_2(\mathcal{N})}{24}. \quad (3)$$

The method here relies on an extension of FHMM combined with the consistency of T^5/Z_2 compactification of M-theory.

In section 2, we review anomaly cancellation for an M5-brane in flat space-time, and then summarize FHMM. We close the section with HMM anomaly computation of A-type theories. In section 3, we generalize to the D-type theory by introducing an orbifold of the form R^5/Z_2 . We propose a simple extension of FHMM to backgrounds with orbifold point, and point out how the anomaly inflow mediated by the antisymmetric tensor charge would be modified. Furthermore, we point out that there is additional inflow associated with the orbifold singularities. We deduce this last contribution from the consistency of the T^5/Z_2 compactification, and compute the anomaly of D-type $(2, 0)$ theories. In section 4, we offer an independent check by estimating the one-loop anomaly contribution from the untwisted sector. It turns out to be consistent with the procedure we adopted, modulo a term that could be cancelled by a six-dimensional local counter-term. In the final section, we discuss a recent conjecture by Intriligator on general form of anomaly of all ADE $(2, 0)$ theories, and close with a few related comments.

2 Anomaly Inflow onto M5 Branes

Let us define an eight-form character I_8 of the space-time curvature R ,

$$I_8 \equiv -\frac{1}{48} \left(p_2 - \frac{p_1^2}{4} \right), \quad (4)$$

where p_n is the n -th Pontryagin character. I_8 appears in numerous context in superstring theories and M-theory, but one directly relevant to us is the topological coupling that exists in M-theory [9],

$$\int C \wedge I_8 = \int G \wedge I_7^{(0)}. \quad (5)$$

C is the 3-form tensor, G is the field strength of C , and $dI_7^{(0)} = I_8$.

Consider an M5 brane in flat space-time. Because M5 is a magnetically charged object,

$$dG = 2\pi\delta_{M5}, \quad (6)$$

there is a six-dimensional gravitational anomaly, generated by variation of the above coupling,

$$-2\pi \int \delta_{M5} \wedge I_6^{(1)} = -2\pi \int_{M5} I_6^{(1)}, \quad (7)$$

with $dI_6^{(1)} = \delta I_7^{(0)}$. On the other hand, a single M5 composed of a single tensor multiplet of $(2, 0)$ supersymmetry. This field content is anomalous and generate one-loop gravitational anomaly of amount

$$2\pi \int_{M5} J_6^{(1)} \quad (8)$$

where $dJ_6^{(1)} = \delta J_7^{(0)}$, $dJ_7^{(0)} = J_8$, and

$$J_8 = -\frac{1}{48} \left(p_2(\mathcal{T}) - \frac{p_1^2(\mathcal{T})}{4} \right). \quad (9)$$

where the characters are those of the tangent bundle \mathcal{T} of the six dimensional world-volume. This anomaly polynomial is equal to I_8 provided that the latter is also evaluated for \mathcal{T} . The resulting anomaly is purely gravitational in six dimensions, and the two clearly cancel each other. We learn that $C \wedge I_8$ is essential in establishing consistency of M5 branes in M theory.

However, not all eleven-dimensional gravitational anomaly would be cancelled this way. Following [7], we split the spacetime curvature R into two parts; one associated

with the six-dimensional tangent bundle \mathcal{T} of M5 and the other associated with the five-dimensional normal bundle \mathcal{N} of M5. On closer inspection, the actual anomaly inflow from $C \wedge I_8$ term corresponds to an anomaly polynomial of the form

$$\begin{aligned} -I_8(\mathcal{T} \oplus \mathcal{N}) &= \frac{1}{48} \left(p_2(\mathcal{T} \oplus \mathcal{N}) - \frac{p_1^2(\mathcal{T} \oplus \mathcal{N})}{4} \right) \\ &= \frac{1}{48} \left(p_2(\mathcal{T}) + p_2(\mathcal{N}) - \frac{(p_1(\mathcal{T}) - p_1(\mathcal{N}))^2}{4} \right). \end{aligned} \quad (10)$$

We used the fact, $p_2(\mathcal{T} \oplus \mathcal{N}) = p_2(\mathcal{T}) + p_2(\mathcal{N}) + p_1(\mathcal{T}) \wedge p_1(\mathcal{N})$. The structure group $SO(5)$ of the normal bundle acts as the R-symmetry group in six-dimensional theory, so the part dependent on \mathcal{N} generates the axial anomaly.

However, the anomaly associated with a single tensor multiplet is found to be,

$$\mathcal{J}_8 \equiv -\frac{1}{48} \left(p_2(\mathcal{T}) - p_2(\mathcal{N}) - \frac{(p_1(\mathcal{T}) - p_1(\mathcal{N}))^2}{4} \right), \quad (11)$$

and we have uncanceled anomaly of the amount [7],

$$-I_8(\mathcal{T} \oplus \mathcal{N}) + \mathcal{J}_8 = \frac{1}{24} p_2(\mathcal{N}), \quad (12)$$

which is purely axial. Since the axial group $SO(5)$ also came from space-time symmetry, its anomaly must cancel in the full M-theory context.

An elegant solution to this puzzle has been proposed by FHMM. They considered a Chern-Simons term,

$$-\frac{2\pi}{6} \int C \wedge dC \wedge dC, \quad (13)$$

argued that this coupling must be modified in a very specific manner in the presence of magnetic sources to C . Introducing the Thom class e_4 for the normal bundle \mathcal{N} of the magnetic source, they smeared out the magnetic source by writing

$$dG = d\rho(r) \wedge e_4/2, \quad (14)$$

where the one-form $d\rho$ is essentially the distribution of the smeared out magnetic source [6]. This allows a simple modification of the above coupling to

$$-\frac{2\pi}{6} \int \tilde{C} \wedge d\tilde{C} \wedge d\tilde{C}, \quad (15)$$

with $\tilde{C} = C - \rho e_3^{(0)}/2$, while the gauge invariant, *everywhere finite*, field strength G is also modified to

$$G = dC - d\rho \wedge e_3^{(0)}/2 = d\tilde{C} + \rho e_4/2. \quad (16)$$

Gauge transformation of $e_3^{(0)}$ induces that of \tilde{C} ,

$$\delta\tilde{C} = -d(\rho e_2^{(1)})/2, \quad (17)$$

which leads to another anomaly inflow,

$$-\frac{2\pi}{6} \delta \int \tilde{C} \wedge d\tilde{C} \wedge d\tilde{C} \quad (18)$$

The integrand is exact, so the integral reduces to the boundary which is the infinitesimal sphere bundle enclosing the M5 brane. Because the boundary involves the infinitesimal sphere, the integral actually reduces to that over M5 brane. Results by Bott and Cattaneo [10] show that the resulting anomaly inflow corresponds to the anomaly polynomial,

$$-\frac{1}{24} p_2(\mathcal{N}). \quad (19)$$

Note that this precisely cancels the remaining anomaly. This mechanism was further studied by Becker and Becker [11] who reconciled it with the cancellation mechanism suggested by Witten [7] in type II perspective, while Ref. [12] made an attempt to study the origin of the function ρ .

Subsequently, Harvey, Minasian, and Moore (HMM) observed [8] that this last anomaly inflow from Chern-Simons term is cubic in the magnetic charge, so the net anomaly inflow onto N coincident M5 branes must be

$$\left(-NI_8(\mathcal{T} \oplus \mathcal{N}) - N^3 \frac{p_2(\mathcal{N})}{24} \right) = - \left(N\mathcal{J}_8 + (N^3 - N) \frac{p_2(\mathcal{N})}{24} \right), \quad (20)$$

negative of which gives the total anomaly polynomial of A_{N-1} (2,0) theory.

3 Anomaly of D-type Theories

To study D-type (2,0) theory, it suffices to put $2N$ coincident M5-branes at origin of R^5 and orbifolding the latter by the parity Z_2 ,

$$x^i \rightarrow -x^i \quad i = 6, 7, 8, 9, 11. \quad (21)$$

This Z_2 has to act on the antisymmetric tensor C as

$$C \rightarrow -C, \quad (22)$$

upon which both topological terms of the previous section remain invariant.

Since this background is not a string theory vacuum, the role of orbifold fixed point is less transparent. Nevertheless, there are several facts known about it. First of all, the orbifold preserves the same set of supersymmetry preserved by M5 branes, so that the world-volume supersymmetry remains $(2, 0)$ on any M5 transverse to R^5/Z_2 . It is also known that the twisted sector fields can be attributed to world-volume degrees of freedom of M5 branes sitting at the singular point [14], which allows us to keep track of the orbifold physics without worrying about how to quantize M-theory.

For the moment, let us first concentrate on anomaly inflow mediated by C field. Early studies taught us that the orbifold fixed point itself has to carry -1 unit of M5 brane charge [15, 14, 16]. After taking this into account, the net M5 brane charge (in the covering space) is $2N - 1$, so the anomaly inflow induced by the M5 brane charge would be¹

$$\frac{1}{2} \left(-(2N - 1)I_8 - (2N - 1)^3 \frac{p_2(\mathcal{N})}{24} \right). \quad (23)$$

The overall factor $1/2$ takes into account the Z_2 modding of R^5 . In the covering space, all that happens is that the field strength G and the normal bundle \mathcal{N} are restricted to respect the orbifolding action. This is more suggestively written as

$$- \left(\left(N - \frac{1}{2} \right) \mathcal{J}_8 + N(2N - 1)(2N - 2) \frac{p_2(\mathcal{N})}{24} \right), \quad (24)$$

where we used the identity (12).

However, this cannot be the end of story. In particular, the correct anomaly inflow must vanish when $N = 0$, since there is no worldvolume anomaly to cancel in that case. Also $N = 1$ should correspond to a free $(2,0)$ tensor theory, whose anomaly is \mathcal{J}_8 . The above inflow is clearly incapable of cancelling this worldvolume anomaly. There must be additional inflow.

A well-known fact is that failure of diffeomorphism invariance may occur at fixed points in a background that is otherwise smooth [3]. In other words, there is an additional anomaly inflow that is associated with the geometry of the orbifold fixed point. Such contribution would have little to do with the topological terms, $C \wedge I_8$ or $\tilde{C} \wedge d\tilde{C} \wedge d\tilde{C}$, and will generate new kind of anomaly inflow associated with the singular nature of the fixed point. One source of anomaly is from the fact that the

¹One may worry that we are smearing out the -1 charge of the fixed point just as we smear out $2N$ charge from M5 brane. However, we find no reason to treat -1 charge differently. For instance, there is a known case, say, in F-theory context, where an orientifold 7-plane is realized as a bound state of 7-branes, each of which are U-dual to D-branes [13].

dimensional reduction of the supergravity is itself anomalous and contributes six-dimensional anomaly localized at the singularities. In the following, we will try to deduce such additional inflow from consistency of an orbifold compactification.

For this, consider anomaly associated with the compactification of M-theory on T^5/Z_2 with 32 fixed points. It has been observed that six-dimensional gravitational anomaly is cancelled satisfactorily if each fixed point carries -1 of M5 brane charge and if we include 16 pairs of M5 branes stuck at half of 32 fixed points [14, 15]. Let us work backward from this knowledge, then, and ask what conclusion we can draw on the contribution associated with the singularity of the fixed point when we include the $SO(5)$ anomaly in the discussion.

Using extension of FHMM adopted above (24), the inflow mediated by C -charge is

$$\frac{1}{2} \left(I_8(\mathcal{T} \oplus \mathcal{N}) + \frac{p_2(\mathcal{N})}{24} \right) = \frac{\mathcal{J}_8}{2}, \quad (25)$$

for each of 16 orbifold points without M5 brane ($N = 0$). For a fixed point with an M5 brane pair ($N = 1$), on the other hand, the world-volume tensor multiplet contributes additional \mathcal{J}_8 , while the anomaly inflow changes the sign,

$$\mathcal{J}_8 + \frac{1}{2} \left(-I_8(\mathcal{T} \oplus \mathcal{N}) - \frac{p_2(\mathcal{N})}{24} \right) = \mathcal{J}_8 - \frac{\mathcal{J}_8}{2} = \frac{\mathcal{J}_8}{2}. \quad (26)$$

Thus, for either kind of fixed point, we find combined anomaly of amount $\mathcal{J}_8/2$ from C -induced inflow and from worldvolume contribution. The net anomaly must be zero, which happens only if further contribution of $-\mathcal{J}_8/2$ exists at each orbifold fixed point.

Anomaly contribution	fixed point	fixed point with a M5-brane pair
Tensor multiplet on M5	0	\mathcal{J}_8
Inflow mediated by C	$\mathcal{J}_8/2$	$-\mathcal{J}_8/2$
Inflow due to singularity	$-\mathcal{J}_8/2$	$-\mathcal{J}_8/2$

Table 1. The first row is due to one-loop contribution from worldvolume fields. The second row is the anomaly inflow from spacetime action involving C -field. Finally, we used anomaly cancellation on the T^5/Z_2 orbifold to determine the last row. See next section for an independent estimate of the last row.

This clearly indicates that we must assign $-\mathcal{J}_8/2$ to each of 32 fixed point, which are all locally of type R^5/Z_2 .

Coming back to N pairs of M5-branes on R^5/Z_2 , this implies that the additional anomaly at origin of R^5/Z_2 should be $-\mathcal{J}_8/2$. Net inflow is then

$$-\left(\left(N - \frac{1}{2}\right) \mathcal{J}_8 + N(2N - 1)(2N - 2) \frac{p_2(\mathcal{N})}{24}\right) - \frac{\mathcal{J}_8}{2}, \quad (27)$$

negative of which should be the anomaly of rank N D-type theory. We conclude that the anomaly of rank N D-type theory is given by

$$N \mathcal{J}_8 + N(2N - 1)(2N - 2) \frac{p_2(\mathcal{N})}{24}. \quad (28)$$

This does reduce to the free tensor theory anomaly when $N = 1$, as it should.

4 Untwisted Sector One-loop Anomaly

However, this is not a rigorous computation. In particular, we have not computed the anomaly inflow due to the singularity; rather we surmised its value, $-\mathcal{J}_8/2$, by relying on the consistency of the T^5/Z_2 compactification. We computed the twisted sector one-loop anomaly and C -induced anomaly, and then inferred what contribution at the singularity is necessary to cancel these. With no particular reason to doubt the consistency of the orbifold in question, any failure of the above procedure would be traced to the fact that we adopted and generalized the FHMM prescription to compute C -induced anomaly inflow, both in the M5 realization of (2,0) theories and in the T^5/Z_2 compactification.

More specifically, there are two potential issues. One is that the FHMM proposal itself involves higher order terms in the M-theory action, which has not been derived independently.² The other, more worrisome issue is whether the orbifolding might induce other changes in the FHMM reasoning that we somehow missed. To the same extent this prescription may be in doubt, so would be the inferred fixed point contribution, $-\mathcal{J}_8/2$, and vice versa. An independent estimate of the latter would go a long way in checking the overall validity of our computation.

Purely gravitational part of $-\mathcal{J}_8/2$, namely $-J_8/2$, was computed directly from the untwisted sector [14, 15]. It arises at one-loop from untwisted sector fields, which

²A different mechanism of anomaly cancellation for a single M5 brane has been suggested [17].

is chiral in the T^5/Z_2 orbifold case.³ We must at least generalize this computation to include axial one-loop anomaly from untwisted sector. One might hope that the required anomaly, $-\mathcal{J}_8/2$, at each fixed point is explained entirely by the one loop contribution. We will see shortly that this is the case up to a (yet to be identified) local counter term.

When one compactifies the 11-dimensional supergravity on T^5/Z_2 , dimensional reduction produces the following field contents: one graviton and fifteen scalars from the metric; four anti-chiral gravitino and twenty chiral spinors from the 11-dimensional gravitino; ten scalars, five chiral 2-forms, and five anti-chiral 2-forms from the 3-tensor C . With respect to the unbroken $(2,0)$ supersymmetry, they group into a supergravity multiplet and five tensor multiplets.

To compute the one-loop anomaly, we also need to know the $SO(5)$ representations under which these untwisted sector fields fall into. With respect to the $SO(5)$ R-symmetry, which is nothing but the rotational group in the compactified direction, the representations are

$$\begin{aligned} g_{\mu\nu} &\rightarrow 1, \\ \psi_\mu^{s(-)} &\rightarrow 4, \\ B_{i\mu\nu}^{(-)} &\rightarrow 5; \end{aligned} \tag{29}$$

$$\begin{aligned} B_{i\mu\nu}^{(+)} &\rightarrow 1 \otimes 5, \\ \psi_i^{s(+)} &\rightarrow 4 \otimes 5, \\ \phi_{ij} &\rightarrow 5 \otimes 5 = (14 \oplus 1) \oplus 10. \end{aligned} \tag{30}$$

In other words, in addition to acting within each supermultiplet as R-symmetry usually does, $SO(5)$ also rotates the five tensor multiplets. This can be seen by observing that chiral and anti-chiral tensors, $B_{i\mu\nu}^{(\pm)}$, combines to form $C_{i\mu\nu}$, or by observing that out of the 25 scalars, $(14+1)$ come from the metric and 10 come from the Hodge dual of C along T^5/Z_2 .

To the one-loop anomaly, contributions from the chiral and anti-chiral tensors

³Since the initial 11-dimensional supergravity is anomaly free, failure of diffeomorphism invariance must localize to orbifold fixed points. A similar phenomenon occurs in the Horava-Witten realization of heterotic string, i.e., M-theory compactified on S^1/Z_2 [3, 4]. The dimensional reduction of 11-dimensional supergravity multiplet produces 10-dimensional chiral supergravity multiplet. Anomaly of the latter is distributed equally at the two fixed points and is cancelled by the twisted sector anomaly from the Yang-Mills multiplets in the two E_8 's.

cancel away, and the nontrivial contribution arises only from fermionic fields. Following Alvarez-Gaume and Witten [18], we find that the anomaly polynomial is the eight-form part of

$$-\frac{1}{2}\mathcal{A}(\mathcal{T}) \wedge (ch_V(\mathcal{T}) - 1) \wedge ch_4(\mathcal{N}) + \frac{1}{2}\mathcal{A}(\mathcal{T}) \wedge ch_{4\otimes 5}(\mathcal{N}), \quad (31)$$

where $ch_{\mathcal{R}}(\mathcal{E})$ is the Chern-character of the bundle \mathcal{E} evaluated on representation \mathcal{R} , and \mathcal{A} is the \mathcal{A} -genus associated with the Dirac operator on spinor. A direct evaluation gives,

$$\frac{1}{3} \left(p_2(\mathcal{T}) - p_2(\mathcal{N}) - \frac{(p_1(\mathcal{T}) - p_1(\mathcal{N}))^2}{4} \right) - \frac{1}{2} p_1(\mathcal{N}) \wedge (p_1(\mathcal{T}) - p_1(\mathcal{N})), \quad (32)$$

which contributes

$$-\frac{1}{2}\mathcal{J}_8 - \frac{1}{64} p_1(\mathcal{N}) \wedge (p_1(\mathcal{T}) - p_1(\mathcal{N})), \quad (33)$$

to each of 32 fixed points.

The first term reproduces *the anomaly inflow due to the singularity* of Table 1.,

$$-\frac{1}{2}\mathcal{J}_8, \quad (34)$$

but we find a disagreement of amount,

$$-\frac{1}{64} p_1(\mathcal{N}) \wedge (p_1(\mathcal{T}) - p_1(\mathcal{N})), \quad (35)$$

from the second term. The discrepancy may not be as bad as it appears, however. It does reproduce both purely gravitational and purely axial part. In particular, the part proportional to $p_2(\mathcal{N})$ is consistent with the previous estimate; we set out to check the validity of the FHMM-like prescription producing the irreducible term, $p_2(\mathcal{N})$, and found that this part of anomaly is reproduced satisfactorily by untwisted sector one-loop computation.

The remaining piece is in a product form and could be cancelled away by a local counter term on the fixed point. A priori, what we computed above is only one specific type of contribution from untwisted sector. Since the discrepancy seems to lie entirely at the orbifold fixed point, we suspect that the failure here has something to do with not taking into full account of the orbifold geometry. One property of the fixed point we have not used properly is that the space R^5/Z_2 has the nontrivial Stieffel-Whitney class w_4 , or equivalently $p_1/2$ considered as an element of the Z_2 cohomology. While this fact is related to the -1 charge of the fixed point, nontrivial

w_4 should have its own physical consequences. It is conceivable that such a twist generates a counter-term, either via a boundary term in the spacetime action or via a mechanism similar to FHMM but associated with curvature.

5 Conclusion

Adopting an extension of FHMM and HMM, and employing the consistency of T^5/Z_2 orbifold of M-theory, we argued that the anomaly polynomial of rank N D-type $(2, 0)$ theory is

$$N\mathcal{J}_8 + N(2N - 1)(2N - 2) \frac{p_2(\mathcal{N})}{24}. \quad (36)$$

An independent check was performed by computing one-loop untwisted sector contribution in M-theory compactified on T^5/Z_2 . Modulo a term cancellable by a local counter-term, it supports the current anomaly estimate.

Recently, K. Intriligator put forward an interesting conjecture [19] that the general anomaly polynomial of the ADE $(2, 0)$ theories takes the form,

$$r\mathcal{J}_8 + g \times c_2 \times \frac{p_2(\mathcal{N})}{24}, \quad (37)$$

where r , g , c_2 are the rank, the dimension, the dual Coxeter number of the respective Lie algebra. As noted by Intriligator, the FMM estimate of the anomaly for A theories indeed agrees with the conjecture. The rank $r = N$ D-type algebra is $so(2N)$, whose dimension is $g = N(2N - 1)$ and whose dual Coxeter number is $c_2 = 2N - 2$, so the result for D-type theories here also agrees with the conjecture. Microscopic origin of order N^3 anomaly [20, 21, 22] has been of considerable interest but still remains mysterious. We hope that one might gain insight from the particular algebraic structure of its coefficient, advocated by Intriligator and confirmed in part here.

An independent confirmation of the result might be available in the AdS/CFT setting. A. Tseytlin [23] attempted to compute sub-leading contributions to conformal anomaly, which is presumably connected to the gravitational and axial anomaly via supersymmetry. His computation revealed no conflict with Ref. [8], but the agreement was somewhat partial since only some of order N terms are computed explicitly. One may hope that a refined version of such computation will give a powerful, albeit indirect, check of axial anomaly computation of both A-type and D-type theories; In the AdS/CFT picture, the dual spacetime backgrounds are devoid of any singularity;

the two cases differ only by the transverse four-manifold being either S^4 or RP^4 . This suggests that no extra difficulty would arise for D-type theories.

Similar computation of anomaly for E-type theories would be desirable, but this case seems much more elusive. No brane picture is known to exist, while a useful AdS/CFT picture is also difficult to come by due to the lack of large N limit. A type IIB picture may work better for a universal approach to all ADE cases, but it remains an open problem.

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